NIRSPEC

UCLA Astrophysics Program **U.C. Berkeley** **W.M. Keck Observatory**

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NIRSPEC Optics Design Note 21.00
Focal Length of TMA

1. Introduction

We need to know the focal length of the TMA for a variety of reasons. First, it determines the plate scale, something which we need to use to translate a physical slit size into a line profile width on the array. Second, it allows us to generally associate output angles on the gratings, and thus wavelength, with location on the array. Last, there is a quality control issue which requires that we estimate the expected focal length of the TMA and compare this to the focal length of the "as-built" TMA.

2. What is Focal Length?

This sounds like a simple question with only one answer, but we can actually define two focal lengths for our purposes. First, we can compare the image size to the object size and use this information to calculate a "magnification effective focal length" as shown in equation (1).

$$\begin{aligned} MEFL &= \frac{h_i}{h_o} EFL_{coll}, \\ \text{where } h_i &= \text{image height}, \\ h_o &= \text{object height}, \\ \text{and } EFL_{coll} &= \text{effective focal length of the collimator}. \end{aligned} \tag{1}$$

This relation seems very simple and useful for calculating the plate scale at the detector; however, it does not necessarily tell us how to relate field angle to location on the array. To do so, we will need another kind of focal length, something I will call the "nominal effective focal length" as shown in equation (2).

$$\begin{aligned} NEFL &= \frac{r}{\theta}, \\ \text{where, } r &= \text{distance the center of the field at the image plane}, \\ \text{and, } \theta &= \text{the field angle with respect to the center of the field}. \end{aligned} \tag{2}$$

The difference between these two definitions for focal length is only nonzero when a system has distortion. An undistorted system will have a constant plate scale, i.e. a constant MEFL, and NEFL will equal MEFL everywhere. For a camera with distortion, the two focal lengths are equal only at the center of the field for small field angles.

3. Anamorphism

Some systems have yet greater complexity and need to be described by separate focal lengths corresponding to orthogonal axes in the focal plane, for instance EFL_x and EFL_y. This property of a system is often referred to as anamorphic magnification, i.e. EFL_x ... EFL_y, and is usually evident for non-axisymmetric optical designs, such as our TMA. With this added twist, we can split equations (1) and (2) into two more equations each, one for each axis in the focal plane.

Unfortunately, we have to add yet still more complexity to our analysis. It turns out that dispersive systems, in general, have intrinsic anamorphic magnification. This comes from the fact that the dispersion only occurs in one plane and not the other. Our system actually has dispersion in two orthogonal planes, one defined by the echelle and one defined by the cross-disperser; however, the amounts of dispersion from each grating, and thus anamorphic magnification, are not equal. We will have to add a factor to equations (1) and (2). I have added this factor to equation (1) for the slit width, w, and for the slit height, h. We can now measure the image size of the slit at various locations on the array, insert these values into equations (3) and (4), and then calculate MEFL of the camera itself. We can also use the measurements to infer NEFL across the format.

$$\begin{aligned} \frac{w_i}{w_o} &= \frac{MEFL_{cam,w}}{MEFL_{coll}} \frac{\cos\theta_i}{\cos\theta_o}, \\ \textit{where } \theta_i &= \textit{the input angle at the echelle}, \\ \textit{and } \theta_o &= \textit{the output angle at the echelle}. \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{h_i}{h_o} &= \frac{MEFL_{cam,h}}{MEFL_{coll}} \frac{\cos\theta_{i,CD}}{\cos\theta_{o,CD}}, \\ \textit{where } \theta_{i,CD} &= \textit{the input angle at the cross-disperser}, \\ \textit{and } \theta_{o,CD} &= \textit{the output angle at the cross-disperser}. \end{aligned} \tag{4}$$

4. Simulations

I wrote a program in the Zemax Programming Language (ZPL) which sends rays from the center of the field through the Keck telescope and NIRSPEC. By varying the wavelength, I was able to probe locations throughout the slit plane. The program prints out the locations and sizes of images at the array. The object and image sizes are compared, and the program solves for MEFL as shown in equations (3) and (4). The echelle is frozen in the simulation, so $\mathbf{2}_i$ = 63.02° for all field points. The output angle, $\mathbf{2}_o$, is calculated from the grating equation, equation (5); in this equation, **(** is the out-of-plane angle, and equals 5° for our system. T is the groove density and is 0.0232 lines/:m for our echelle grating.

$$m\lambda T = (\sin\theta_i + \sin\theta_o)\cos\gamma . \qquad \textbf{(5)}$$

The cross-disperser grating is also frozen in the simulation, and it produces the same type of effect as described in equation (5). The cross-disperser has a groove density of 0.075 lines/:m and is used with (= 0°.

Figure 1 gives a map of MEFL across the format. The orders stretch up and down in this picture with increasing wavelength going up, and the cross-disperser spreads light left and right with higher wavelengths to the left. The points locate positions on the array where the left and bottom axes of the plot list pixel number from the center of the format. We can see from the plot that the TMA camera by itself would stretch objects more in the vertical direction, i.e. the focal lengths above each point tend to be larger than the focal lengths to the left of each point. Of course, we have to remember that this map of the camera magnification is only part of the story. Recall that the gratings themselves produce varying magnification across the format. It turns out that the variation in camera magnification throughout a spectral order is compensated by the variation in magnification induced by the gratings. So, in testing the TMA by itself, we should expect to see about ±8% variation in MEFL along the vertical direction and about half as much variation along the horizontal direction.

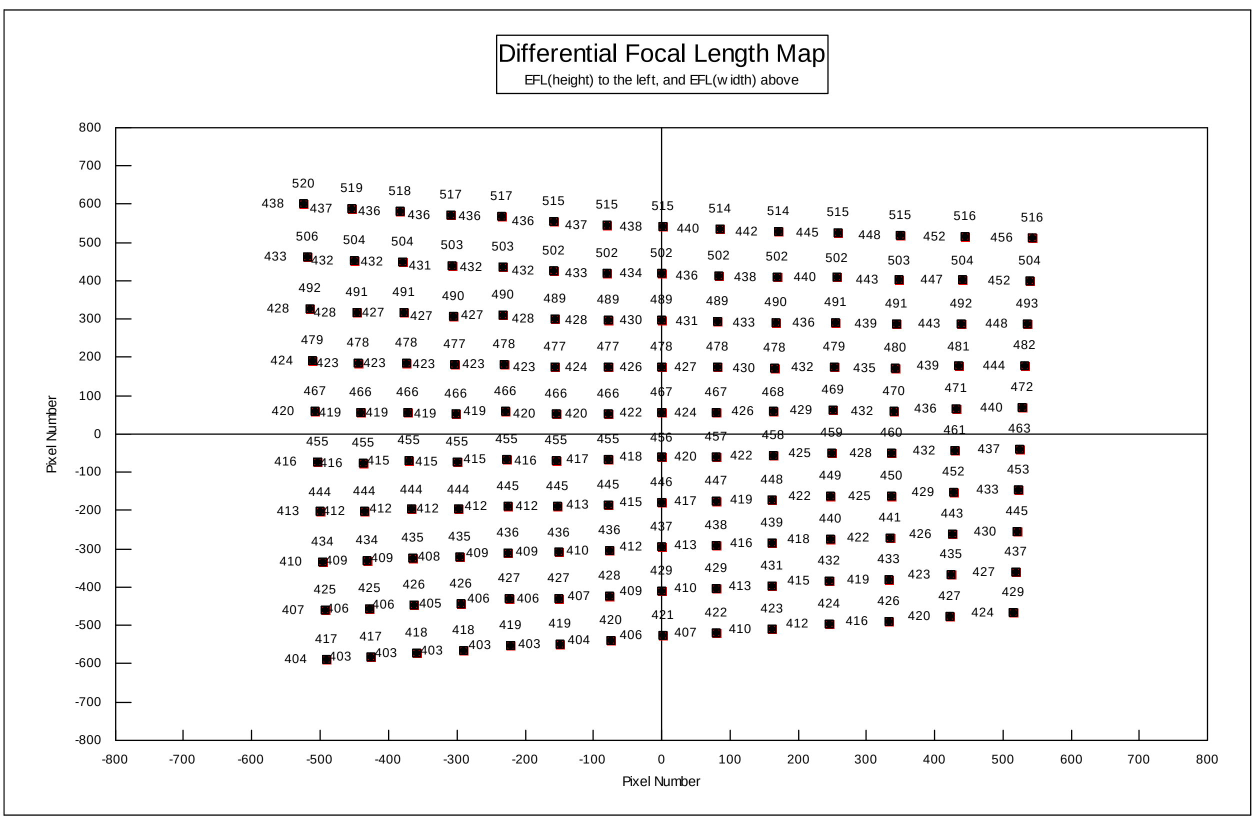

Figure 1.

Just for completeness, we can plot the actual plate scale along both axes in order to visualze the combined effects of the gratings and the TMA distortion. This is shown in Figure 2.

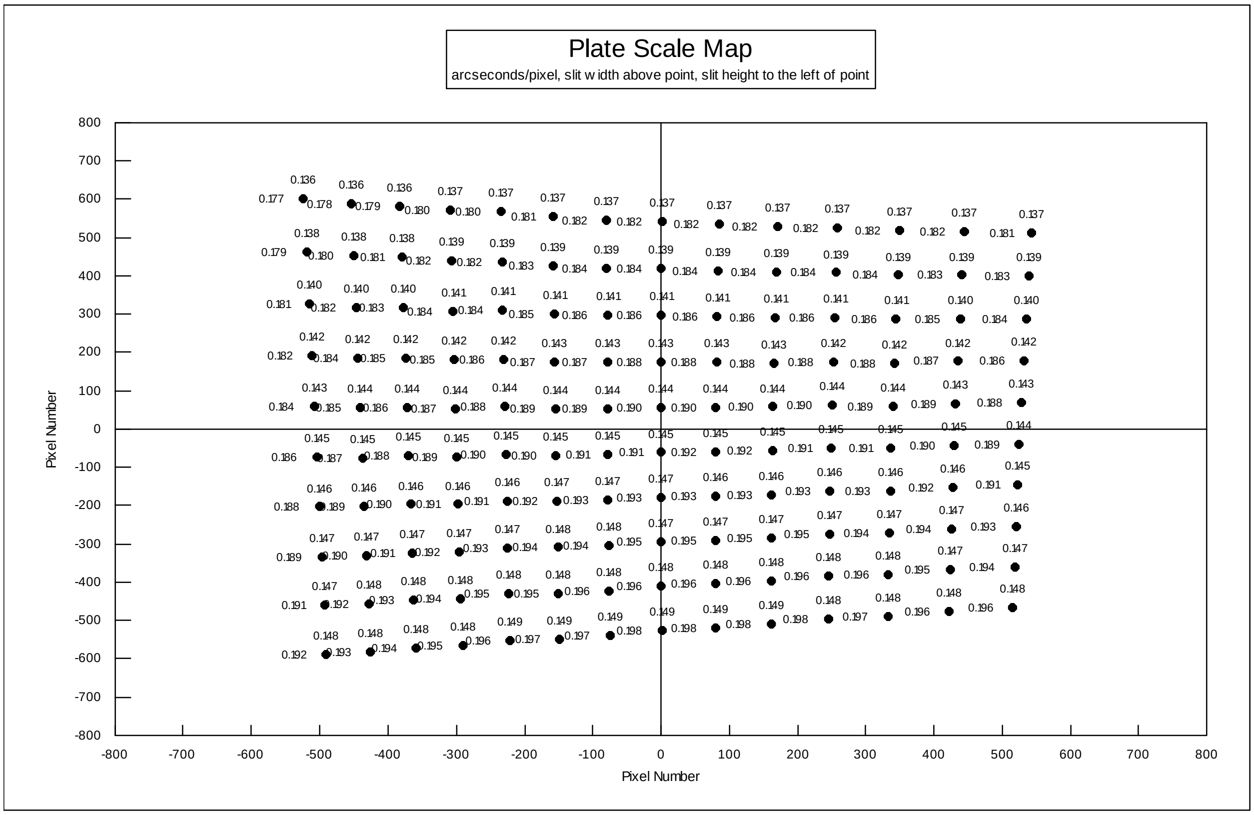

Figure 2.

As we stated earlier, MEFL is useful for calculating the plate scale at a given field point, but we will need to determine NEFL in order to associate a location on the array with input angle in the camera. This sounds easy at first glance, but there is yet another complication. In equation (2), $\mathbf{2}$ is measured with respect to the center of the field. When we split the definition along the two axes, $\mathbf{2}$ should be measured as the angular distance from the appropriate center line. So, for instance, $\mathbf{2}$ and r would be measured with respect to the horizontal or vertical lines in Figures 1 and 2, depending on whether we are interested in $NEFL_w$ or $NEFL_h$. The new complication arises because there is no single output angle which maps into one of the two axes in Figures 1 and 2. We might expect that $\mathbf{2}_o = 63.02°$ should map to the horizontal axis in the figures for all $\mathbf{2}_{o,CD}$. Unfortunately, this is not the case. Why? Because the TMA induces a distortion which will map a straight line into a curved one. So, this means that we must determine the output angle which corresponds to each axis on the array at all positions along the array. Then we can subtract this angle from the output angle of each field point in order to calculate the "$\mathbf{2}$" in equation (2). To give a concrete example, I have calculated that the center line in the figures is covered by output angles which smoothly range from 62.91° to 63.19° going from right to left in the figures. I have calculated these axis-crossing output angles for each echelle order. With this, we can calculate $NEFL_w$ and $NEFL_h$ as shown in equations (6) and (7). The results of this analysis are shown in Figure 3. Here we can see that the NEFL values are nearly the same as the magnification focal lengths given above.

$$\begin{aligned} NEFL_w &= \frac{y}{\theta_o - \theta_{o,0}}, \\ \textit{where } y &= \textit{image distance horizontal centerline on the array}, \\ \theta_o &= \textit{output angle at echelle}, \\ \textit{and } \theta_{o,0} &= \textit{output angle at echelle for field point which is mapped onto the centerline}. \end{aligned} \qquad \textbf{(6)}$$

$$\begin{aligned} NEFL_h &= \frac{x}{\theta_{o,CD} - \theta_{o,CD,0}}, \\ \textit{where } x &= \textit{image distance vertical centerline on the array}, \\ \theta_{o,CD} &= \textit{output angle at cross-disperser}, \\ \textit{and } \theta_{o,CD,0} &= \textit{output angle at cross-disperser for field point} \\ & \quad \textit{which is mapped onto the centerline}. \end{aligned} \qquad \textbf{(7)}$$

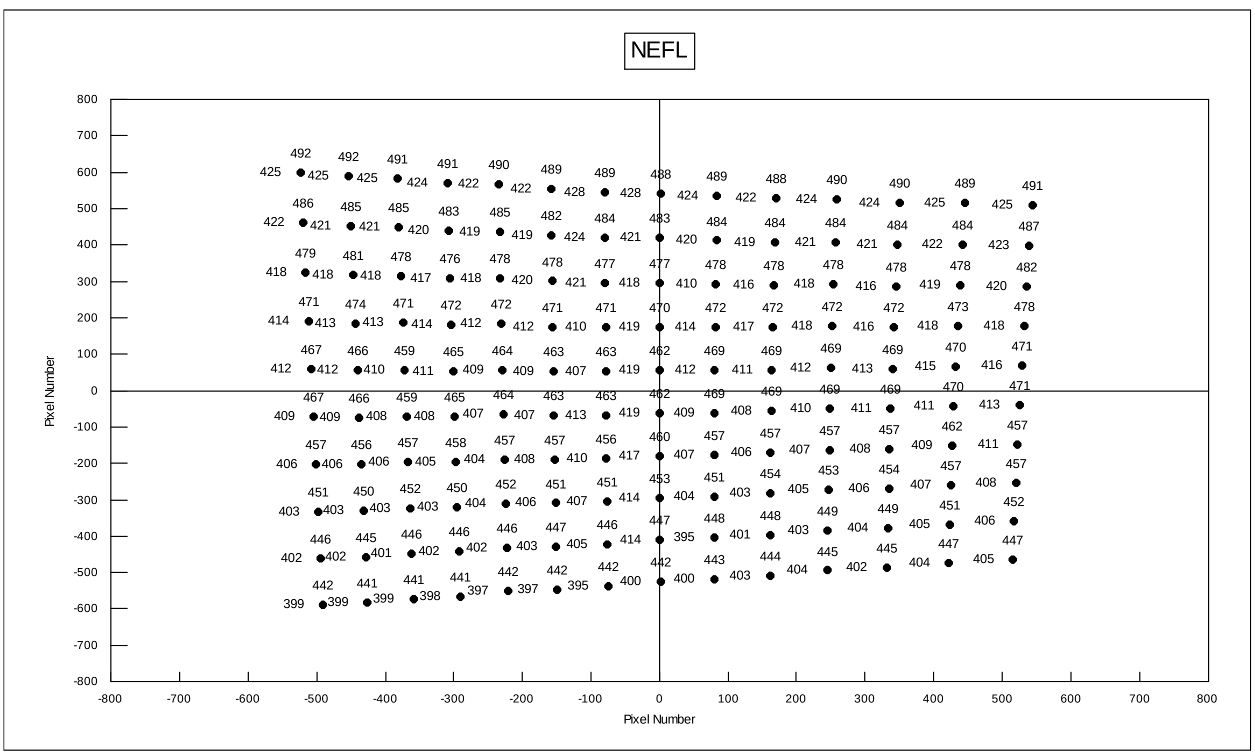

Figure 3.

It is useful to know the zero point angles which were used to calculate NEFL above. With these numbers, we can determine the true location of a field point on the array taking into account distortion. I have plotted the zero point angles for the echelle and cross-disperser as a function of pixel number on the array. For example, the zero point angle changes along the horizontal axis of the array, and this is plotted in Figure 4. I have plotted the same thing in Figure 5, except for the cross-disperser. In this case, the "output angle" really refers to the angle between the input and output rays. The nominal value of 50° is supposed to correspond to the vertical axis on the array, but we can see again that the TMA is distorted.

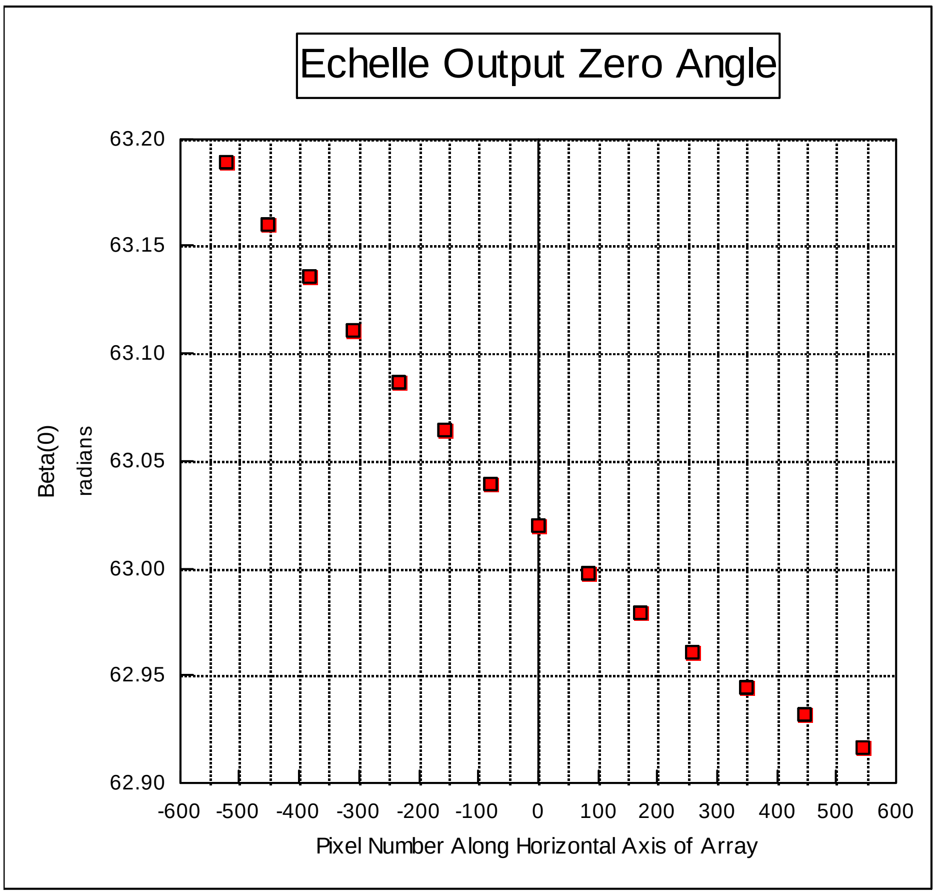

Figure 4.

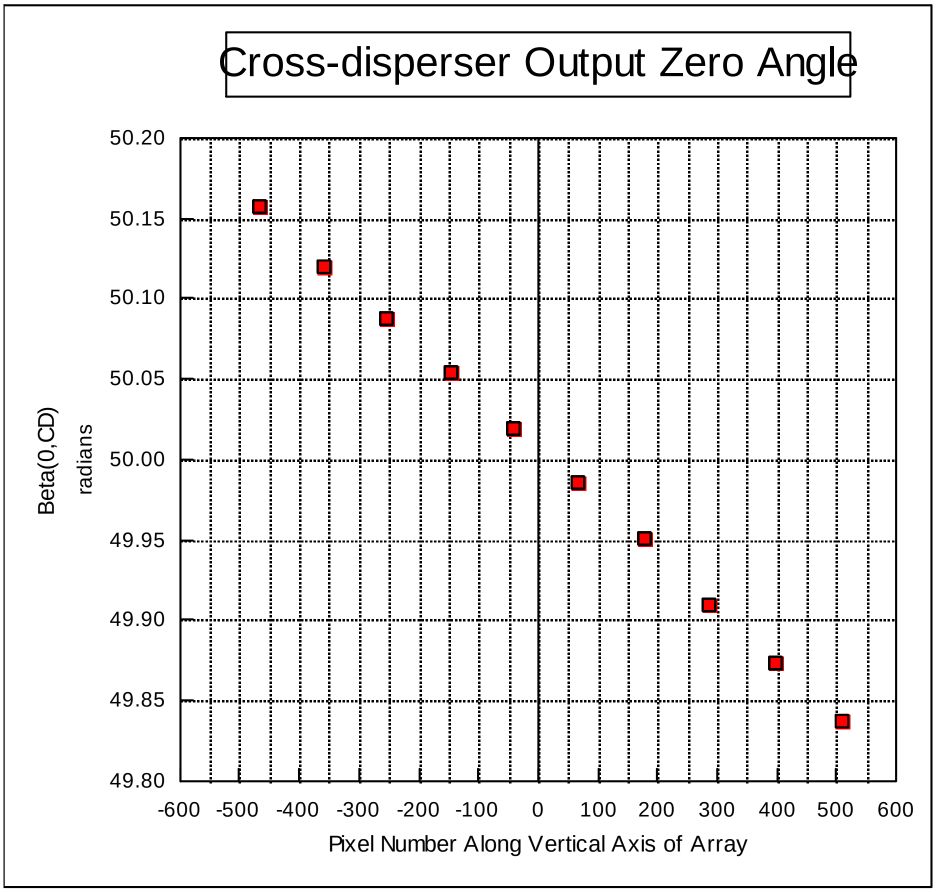
Cross-disperser Output Zero Angle
Beta(0,CD)
radians
50.20
50.15
50.10
50.05
50.00
49.95
49.90
49.85
49.80
-600 -500 -400 -300 -200 -100 0 100 200 300 400 500 600
Pixel Number Along Vertical Axis of Array

Figure 5.